Sanskardham Kelavani Mandal’s

# Jashbhai Maganbhai Patel College Of Commerce

**NAAC RE-ACCREDITED, GRADE-C, CGPA-1.85**

Off. M.G. Road, Near Azad Maidan , Goregaon (West).

## Department of Information Technology

**PROGRAMME: F.Y.B.Sc.I.T.**

**SEMESTER: I**

**COURSE:** Computational Logic and Discrete Structure

Ms. Ansari Aayesha

# Topic: Set Theory



- What is set
- How the elements will represents
- How to enclose the elements
- How to denote the set name
- How to label the elements of sets
- Types of sets
- Operations on sets

# Set theory is a branch of mathematical logic that studies SET



**Set :** A collection of well defined object is known as **SET.**

The modern study of set theory was initiated by George cantor & Richard Dedekind in 1870s.

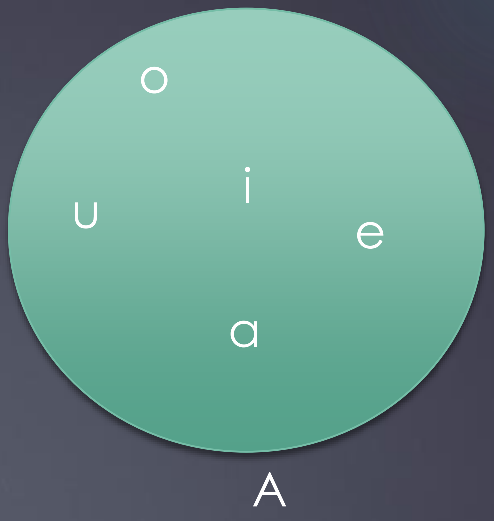
After the discovery of paradoxes in naive set theory, such as Russell's paradox numerous axioms systems were proposed in the early twentieth century.

**Set :** A collection of well defined object is known as **SET.**

- Collection of vowel,
- Collection of states
- Collection of letters of word " **COMPUTER** ".
- Sets elements are enclosed in curly brackets { }
- A= {a, e, i, o, u}
- Set name A
- Elements of set A: a, e, i, o, u

Elements of the set are the object contained in set

- A= {a, e, i, o, u}




a $\epsilon$ A  a belongs to set A
o $\epsilon$ A  o belongs to set A

p $\epsilon$ A it is wrong – false
P $\notin A$  p does not belongs to set A

**NOTE:**

Set name is always denoted by capital letters

Elements of sets are usually denoted by small letters

Representation of set

Roster method

Set Builder method

# Roster method

- This method is also called Tabular method / listing method
- In this method all the elements are listed separated by commas within { }
- Eg. The Collection of Vowels in word "Independence"

$$\therefore X = \{ i, e \}$$

- The order of writing the elements of set is immaterial
- An elements of a set is not written more than once.

# Set builder method



- This is also called as property form

- It is represented by stating all the properties **p(x)** which are satisfies by the element **x** of the set and not by the other elements outside the set

1. If A is set of even natural numbers

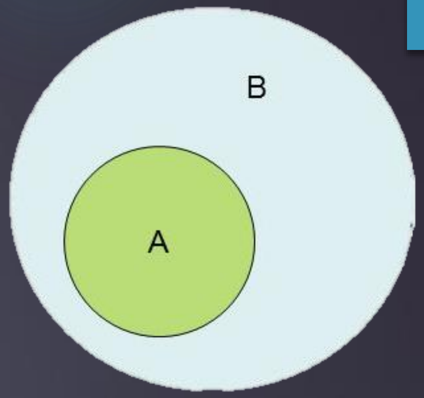
   $A = \{ x : x = 2n , x \in \mathbb{N} \}$

2. If B is set of odd numbers

   $B = \{y : y = 2n+1 \}$

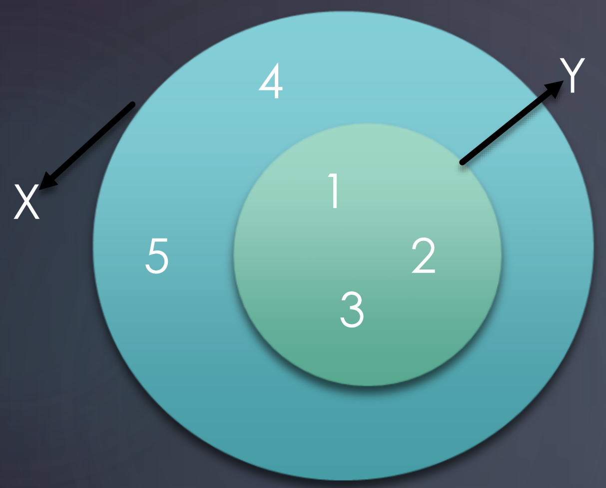
# ❖ SUBSET $\subseteq$

Let A and B are two non-empty sets, Then set A is said to be subset of set B if and only if all elements of set A are in set B.




$A \subseteq B$

**$A \subseteq B$ iff $\forall x$ , if $x \in A$ then $x \in B$**




$Y \subseteq X$

$X = \{1,2,3,4,5\}$

$Y = \{1,2,3\}$

All elements of Y are belongs to set X

$1 \in Y \Longrightarrow 1 \in X$

$2 \in Y \Longrightarrow 2 \in X$

$3 \in Y \Longrightarrow 3 \in X$

Therefore Y is subset of X i.e. **$Y \subseteq X$**

**On other hand X is not subset of Y**

**as $5 \in X$ but $5 \notin Y$**

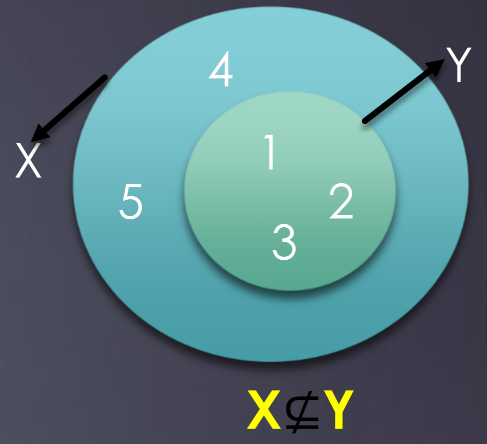
**$\therefore X \nsubseteq Y$ i.e. X is not subset of Y**

- $A \subseteq A$
- **A is subset if itself always.**
- **$A \not\subseteq B$ iff $\exists\, x \in A$ such that $x \notin B$**

In given diagram

**$X \not\subseteq Y$**

as $4 \in$ **X but 4** $\notin Y$




❖ Check which of the following are subset of which other set

X= { 1, 2, 3, 4, 5 , 6, 7}

Y = { 7, 8, 9, 10}

Z= { 2, 4, 6}

E= {8, 9}

S= {7}

# ❖ PROPER SUBSET $\subset$

If set A is subset of set B and is not equal to set B then we called it as A is proper subset of B , and it is denoted as

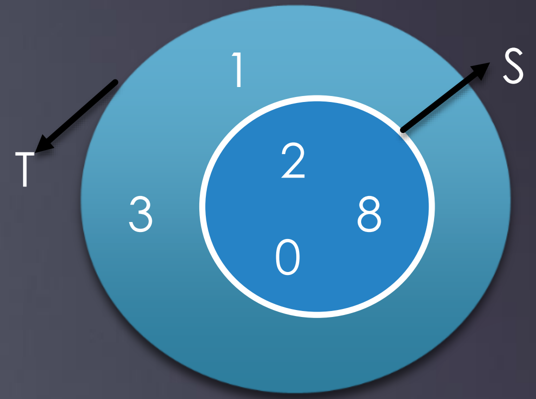
$$A \subset B$$

Let T= { 1, 2, 3,8, 0}

S = { 2, 8, 0}

$S \subseteq T$ and $S \neq T$

$$\therefore S \subset T$$




$S \subset T$

**NOTE:** $S \not\subset S$

A : Collection of letters of word “EAT”


B : Collection of letters of word “TEA”


$A \subseteq B$ and $A = B$
$\therefore A \not\subset B$

P : Collection of letters of word “EAT”


Q: Collection of letters of word “TEAM”


$P \subseteq Q$ and $P \neq Q$
(as $m \in Q$ but $\mathbf{m} \notin P$
Q has one extra element as compare to P)
$\therefore P \subset Q$

# ❖ EQUALITY OF SET

Two sets are said to be equal if and only if every element of one set is contain in other set and vice versa.

Set A is said to be equal to set B
If and only if (iff) every element of set A is in set B and every element of set B is in set A

$$\mathbf{A = B \Leftrightarrow A \subseteq B \text{ and } B \subseteq A}$$

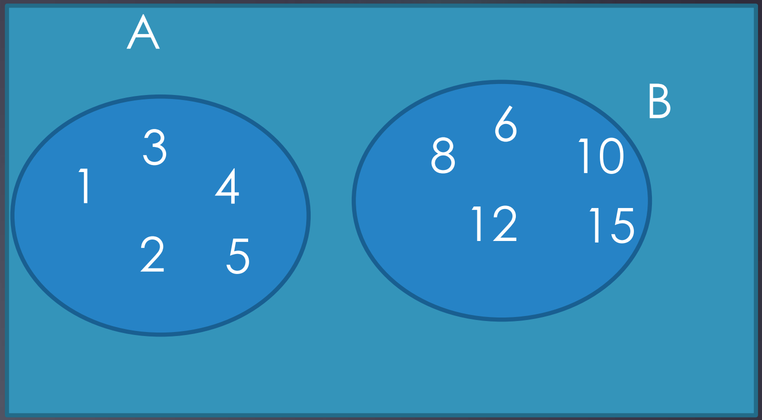
Let A: collection of letters of word " TEN"
Let B : Collection of letters of word "NET"
A = B
Since A= {t, e, n} , B= { n, e, t}
Hence A = B

# ❖ UNION OF SET : U



Union of set A and B is collection of all elements of set A and set B without

**AUB= {x : x ϵ B or x ϵ A }**




**A U B**

A= {1,2,3,4,5}

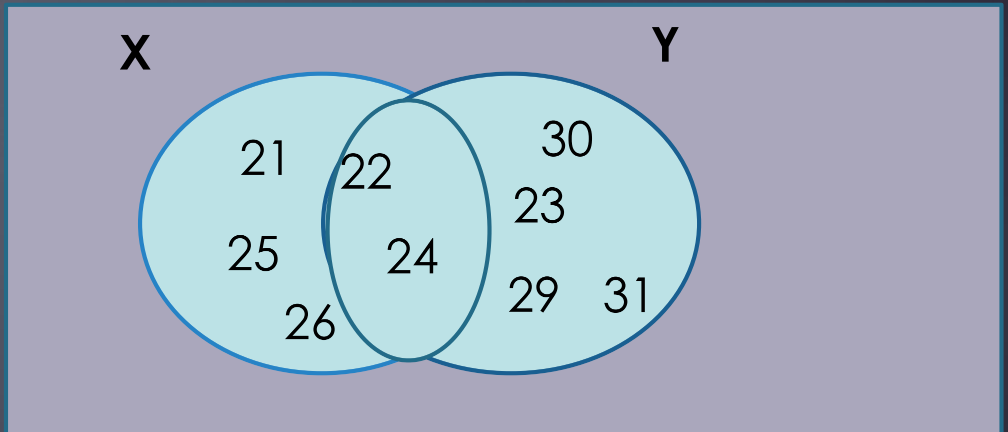
B={6,8,10,12,15}

A U B = {1,2,3,4,5,6,8,10,12,15}

X= {21,22,24,25,26}

Y= {22,23,24,29,30,31}

XUY= {21,22,23,24,25,26,29,30,31}

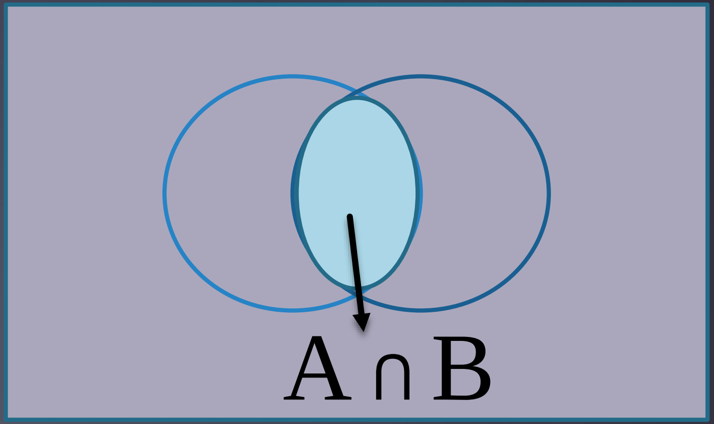



**X U Y**

# INTERSECTION OF SET : ∩

Intersection of set containing a common elements of sets.

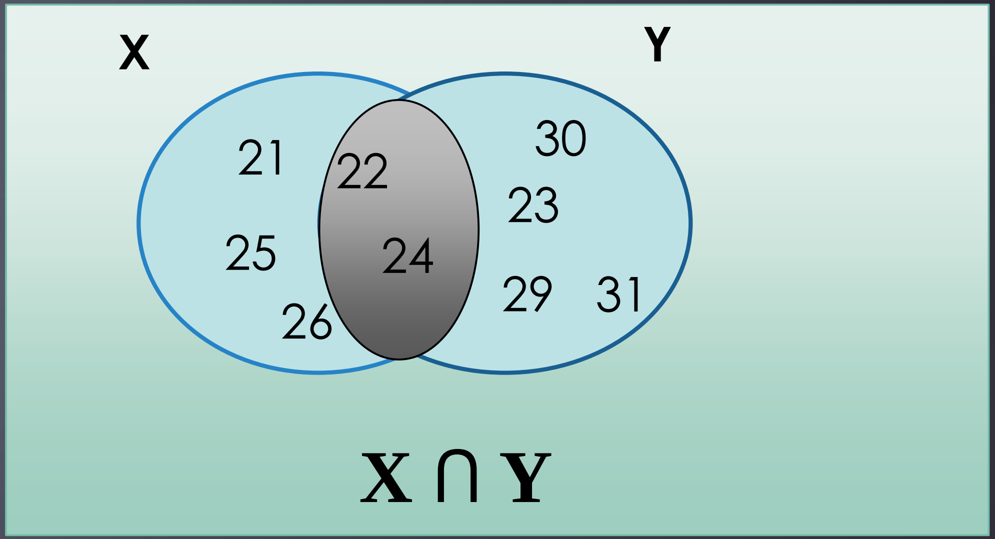
$$A \cap B = \{ x : x \in A \text{ and } x \in B \}$$




X= {21,22,24,25,26}

Y= {22,23,24,29,30,31}

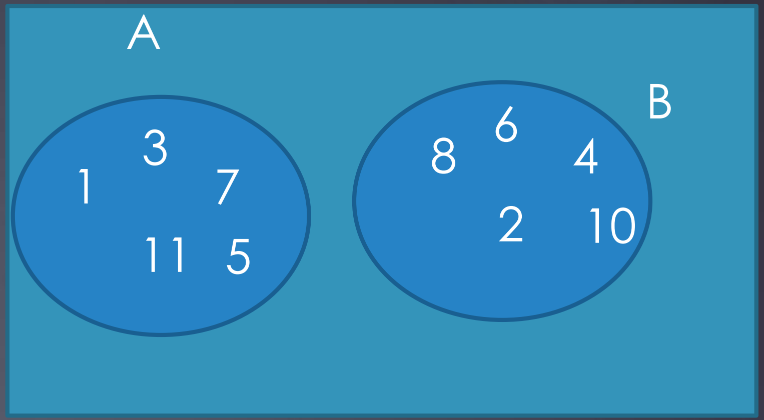
X ∩ Y= {22,24}



A= {1,2,3,4,5}

B={2,4,5,6,8}

A ∩ B = { } - Null set or Empty set



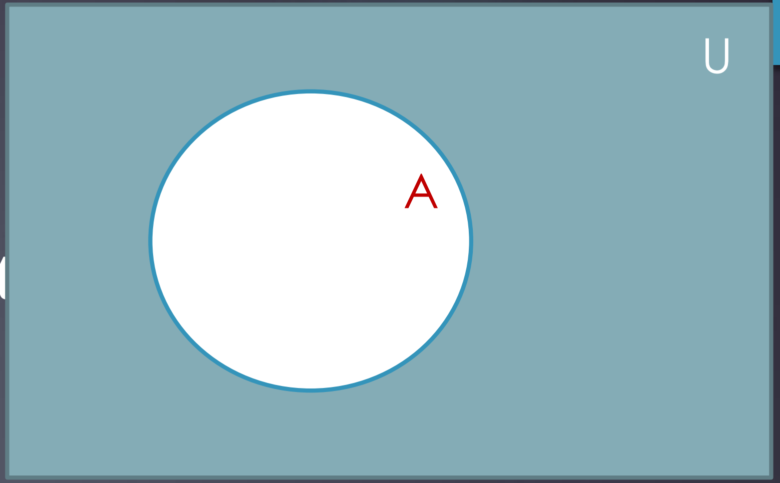

**A ∩ B**

# ❖ COMPLEMENT OF SET

Complement of set A is collection of all those element which are not present in A but in universal set.

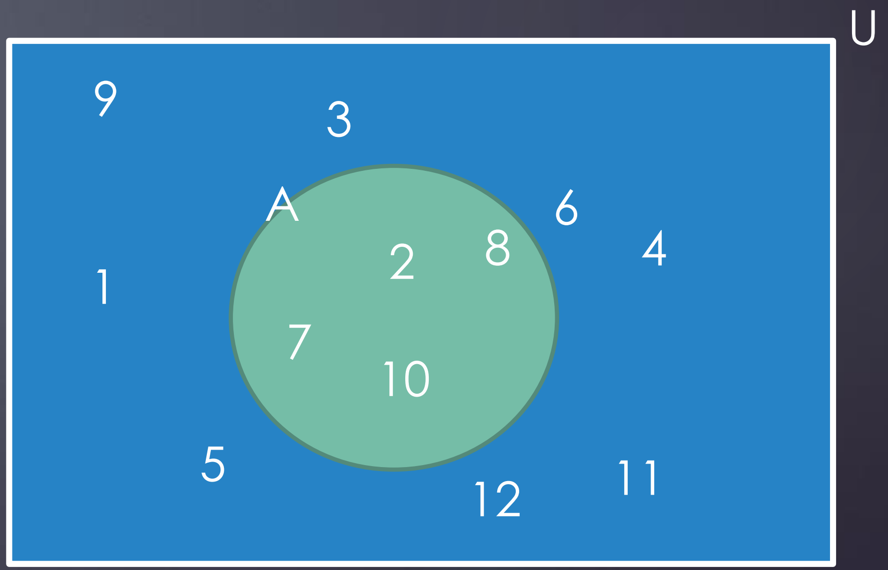
It is denoted by A’ or $A^c$

$$A^c = \{ x : x \in U \text{ but } x \notin A \}$$




Shaded region is complement of set A

$A^c$




$U = \{ 1,2,3,4,5,6,7,8,9,10,11,12\}$

$A = \{2,7,8,10\}$

$A^c = \{ 1,3,4,5,6,9,11,12\}$

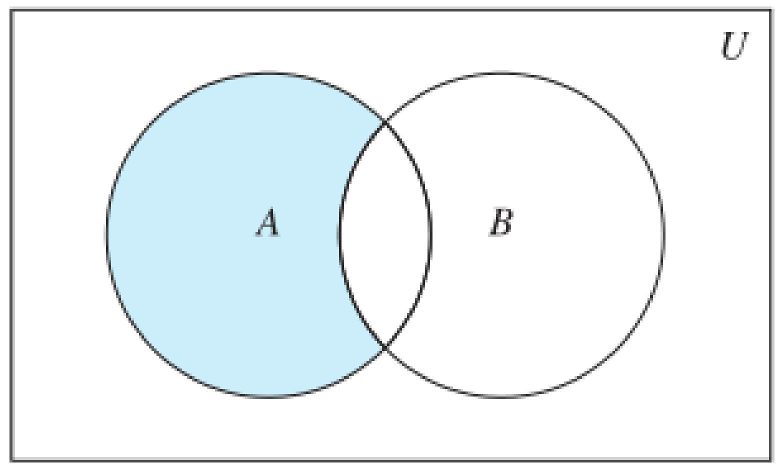
# ❖ DIFFERENCE BETWEEN SET

The difference between set A to set B is collection of all those elements which are present in set A but not in set B

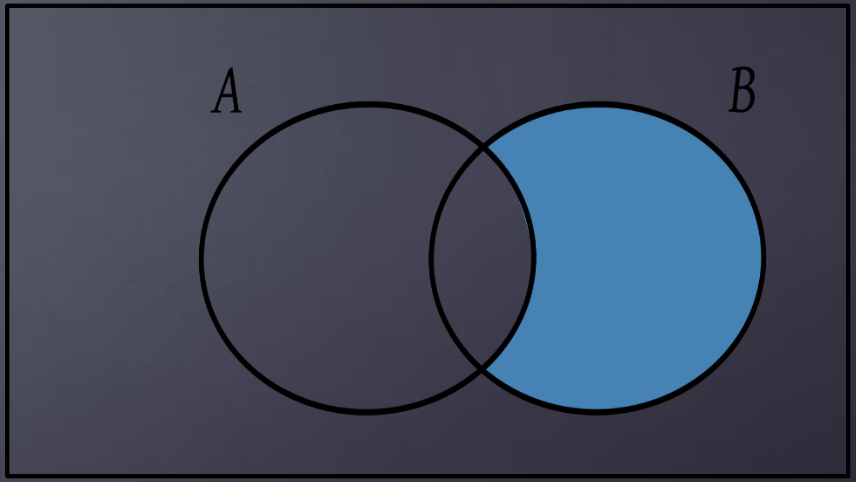
**A - B = { x : x ϵ A but x ∉ B }**

**B - A = { x : x ϵ B but x ∉ A }**

- A – B is also denoted as A \ B




A − B is shaded.




Shaded region is B- A

J.M.PATEL COLLEGE OF COMMERCE

Ms. Ansari Aayesha

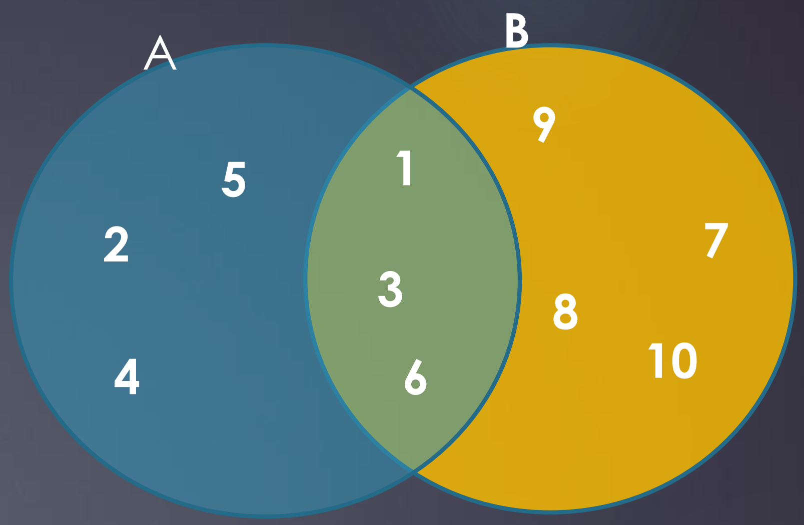
Let A = { 1,2,3,4,5,6}

B= {1,3,6,7,8,9,10}

**Then A-B = {2,4,5}**

Similarly

**B-A ={7,8,9,10}**




Blue shaded is A-B
And
Yellow shad is B-A

# CARTESIAN PRODUCT OF SET



Cartesian product of set A to set B is defined as
A collection of an ordered pair of an elements of

**A X B = { (x, y) : x ϵ A and y ϵ B }**

Let A = {1,2,3} , B= { 2,4,5} Then ,
A X B = { (1, 2), (**1**, 4), (**1**, 5), (**2**, 2), (**2**, 4), (**2**, 5), (**3**, 2), (**3**, 4), (**3**, 5)

P= { 2, 8, 9} , Q= {4, 10}

1. P X P = { (2,2), (2,8), (2,9), (8,2), (8,8), (8,9),(9,2), (9,8), (9,9)
2. P X Q = { (2, 4), (2,10), (8, 4), (8, 10), (9, 4), (9,10) }
3. Q X P = { (4, 2), (4, 8), (4, 9), (10, 2), (10, 8), ( 10,9) }
4. Q X Q = { (4,4,), (4, 10), (10, 4 ), (10, 10 ) }

## ❖ PROPERTIES OF SET



| | |
|---|---|
| 1. Inclusion of Intersection | a. $A \cap B \subseteq A$<br>b. $A \cap B \subseteq B$ |
| 2. Inclusion in Union | a. $A \subseteq A \cup B$<br>b. $B \subseteq A \cup B$ |
| 3. Transitive property of subset | If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$ |
| 4. Commutative Laws: | a. $A \cup B = B \cup A$<br>b. $A \cap B = B \cap A$ |
| 5. Associative Law | a. $A \cup (B \cup C) = (A \cup B)\ C$<br>b. $A \cap (B \cap C) = (A \cap B) \cap C$ |
| 6. Distributive law | a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| 7. Identity law | a. $A \cup \emptyset = A$<br>b. $A \cap U = A$ |
| 8. Complement Law | a. $A \cup A^c = U$<br>b. $A \cap A^c = \emptyset$ |

Ms. Ansari Aayesha

# ❖ PROPERTIES OF SET

| | |
|---|---|
| 9. Double Complement Law | a. $(A^c)^c = A$ |
| 10. Idempotent Law | a. $A \cup A = A$<br>b. $A \cap A = A$ |
| 11. Universal bound laws | a. $A \cup U = U$<br>b. $A \cap \emptyset = \emptyset$ |
| 12. De-Morgan's law | a. $(A \cup B)^c = A^c \cap B^c$<br>b. $(A \cap B)^c = A^c \cup B^c$ |
| 13. Absorption Law | a. $A \cup (A \cap B) = A$<br>b. $A \cap (A \cup B) = A$ |
| 14. Complement of Universal and empty set | a. $U^c = \emptyset$<br>b. $\emptyset^c = U$ |
| 15. Set Difference Law | a. $A - B = A \cap B^c$ |

Out of 80 players 35 play cricket, 42 play hockey, 25 play football, 8 play both cricket and hockey, 10 play both cricket and football, 7 play both hockey and football, 3 play all the three games.

1) Find how many play only cricket
2) How many play exactly hockey and football
3) How many playing only one game?

Let C: players playing Cricket

H: players playing Hockey

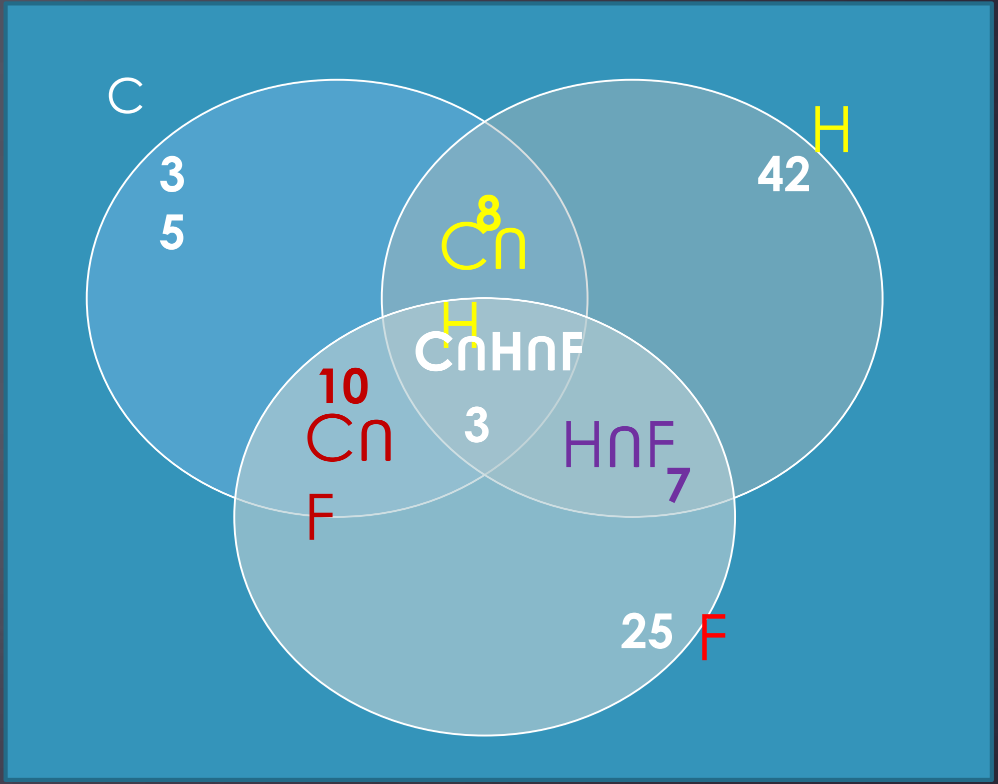
F : players playing Football

C∩H : player playing both the games cricket and hockey

C∩F : players playing both the games cricket and football

F∩H : players playing both the games football and hockey

C∩H∩F : players play all the three games cricket, hockey and football




Ms. Ansari Aayesha

$|C| = 35$

$|H| = 42$

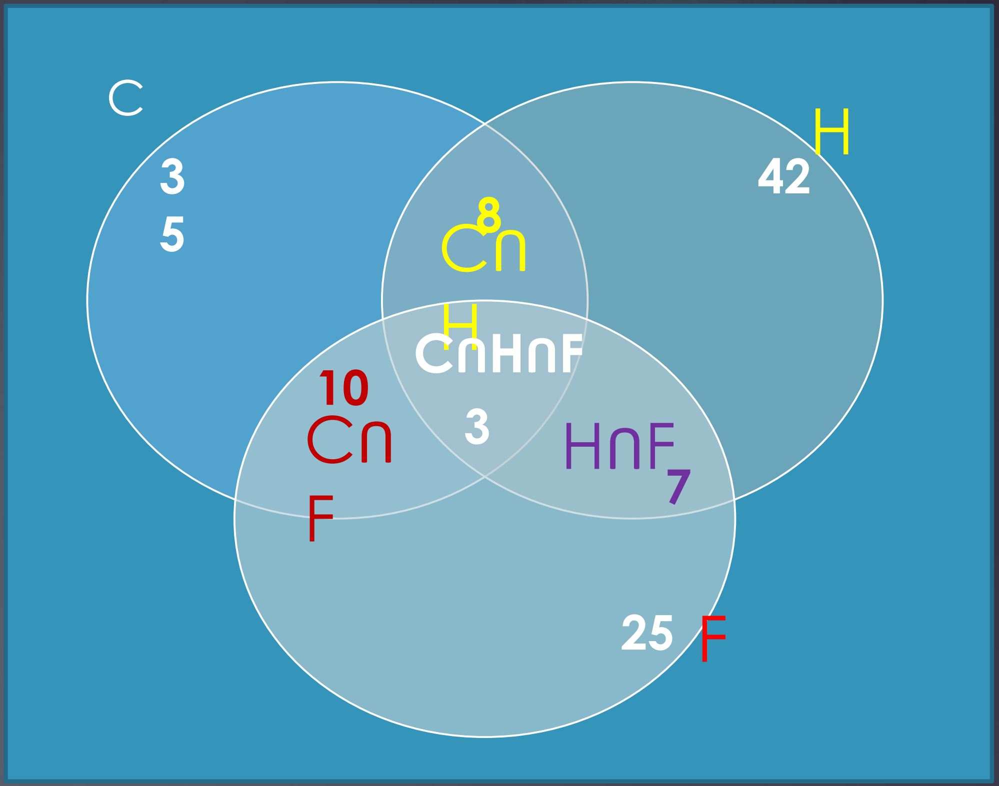
$|F|=25$

$|C \cap H| = 8$

$|C \cap F|= 10$

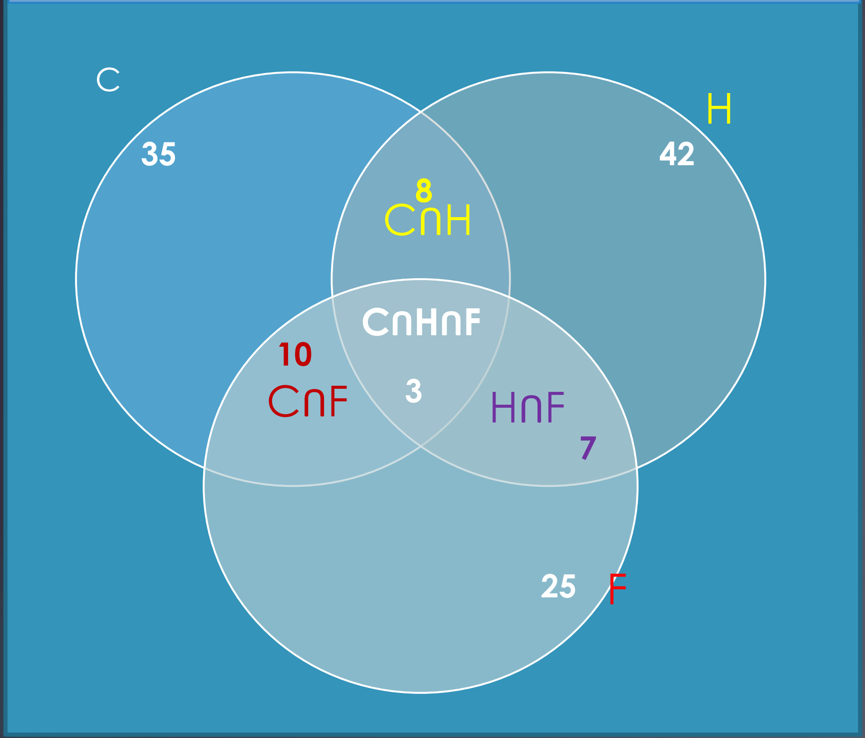
$|H \cap F| = 7$

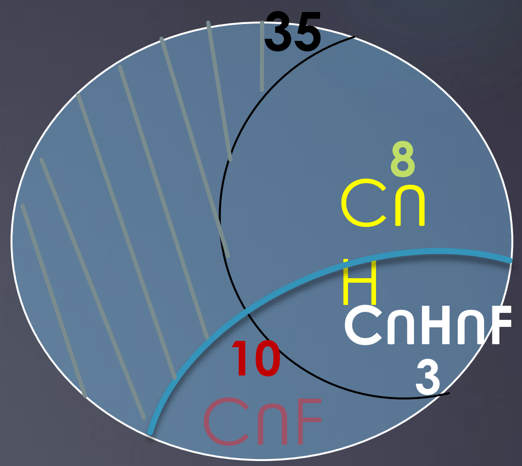
$|C \cap H \cap F|= 3$




Ms. Ansari Aayesha

1) Find how many play only cricket




Shaded lined region represent :players playing only cricket

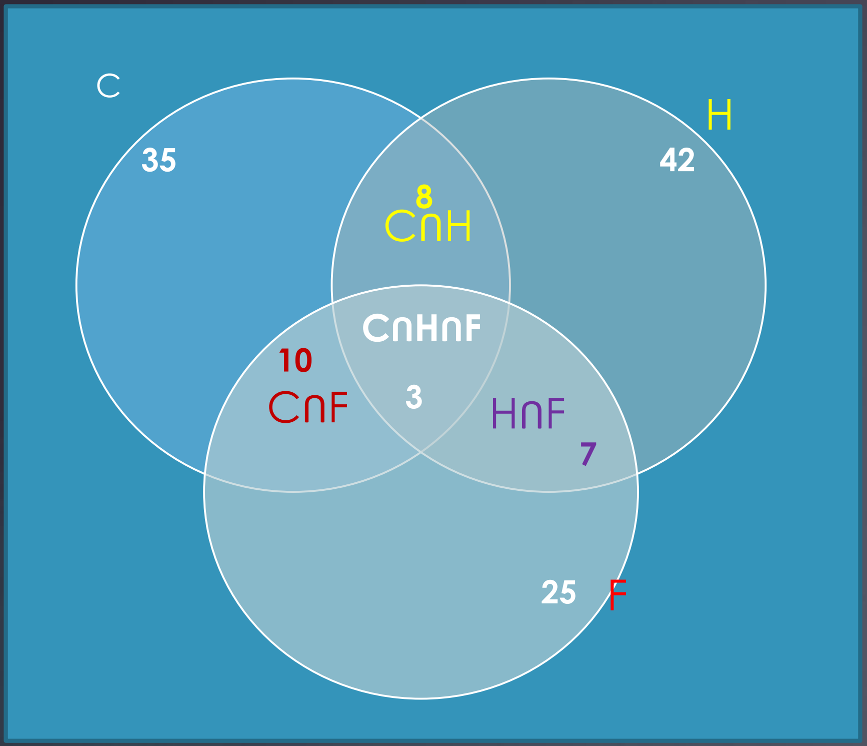
$|player\ playing\ only\ cricket|$ =$|C|$ - $|C \cap H|$-$|C \cap F|$ + $|C \cap H \cap F|$

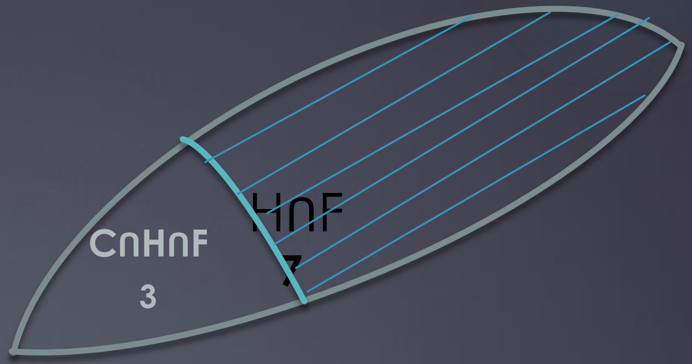
$|player\ playing\ only\ cricket|$ =$35 - 8 - 10 + 3$

$|player\ playing\ only\ cricket|$ =20

Ms. Ansari Aayesha

## 2) How many play exactly hockey and football




Shaded lined region represent :players playing exactly Hockey and Football

$$|Player\ playing\ exactly\ hockey\ and\ football| = |H \cap F| - |C \cap H \cap F|$$

$$|Player\ playing\ exactly\ hockey\ and\ football| = 7 - 3$$

$$|Player\ playing\ exactly\ hockey\ and\ football| = 4$$

Professor in a discrete mathematics class passes out a form asking students to check all the mathematics and computer science courses they have recently taken. The finding is that out of a total of 50 students in the class, 30 took precalculus; 16 took both precalculus and Java; 18 took calculus; 8 took both calculus and Java; 26 took Java; 47 took at least one of the three courses. 9 took both precalculus and calculus;

a. How many students did not take any of the three courses?

b. How many students took all three courses?

c. How many students took precalculus and calculus but not Java?

d. How many students took precalculus but neither calculus nor Java?